INFORMATION AND STRATEGIC VOTING

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ABSTRACT
We theoretically and experimentally study voter behavior in a setting characterized by plurality rule and mandatory voting, where voters choose from three options. We are interested in the occurrence of strategic voting in an environment where Condorcet cycles may occur. In particular, we focus on how information about the distribution of preferences affects strategic behavior. We also vary the relative importance of the second preferred option to investigate how this affects the strategic vote. Quantal response equilibrium analysis is used to analyze the game and proves to be a good predictor for the experimental data. Our results indeed show that strategic voting arises, the extent of which depends on (i) the availability of information; (ii) the relative importance of the intermediate candidate; (iii) the electorate’s relative support for one’s preferred candidate; and (iv) the relative position of the plurality-supported candidate in a voter’s preference ordering. Our results show that information serves as a coordination device where strategic voting does not harm the plurality-preferred candidate’s chances of winning.

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1 Introduction

Since its introduction in ancient Greece, democracy has always been associated with ‘government by the people’. A widespread view is that the democratic decision process must honor the desire of the majority (Goldfinger 2004).\(^1\) Voting is the tool most often used for this purpose. The underlying assumption is that voting correctly aggregates individual preferences. In most democratic countries, we vote at almost every level of social interaction: at faculty meetings; professional organizations; shareholder gatherings; and in national referenda or elections.\(^2\) A sufficient condition for correct aggregation of preferences is that every voter casts a vote for her most preferred alternative.\(^3\) Of course, not everyone does so. For one thing, many people abstain from voting (especially in large scale elections). If abstention is correlated with preferences, the preferences represented by the votes cast no longer mirror those of the electorate (Großer and Schram 2010). For example, Lijphart (1997) argues that low turnout in U.S. elections yields significant underrepresentation of the interests of “less well-to-do citizens”. Moreover, voters may strategically vote for an alternative that is not ranked highest in their preference ordering (Farquharson, 1969). The reason is that any election is not only a manifestation of individual preferences, but also a multi-person decision process (Downs 1957; Riker 1982a; Blais and Nadeau 1996). In such a strategic interaction a voter may be more interested in optimizing the outcome than in stating her own preference.\(^4\)

When considering voting as a multi-person decision process it can be analyzed as a strategic game in which distinct strategies might lead to different outcomes and equilibria can be computed. It has long been recognized that strategic voting may be an equilibrium strategy in committees (Austen-Smith and Banks 1996), legislatures (Riker 1982a) and even in large electorates (Palfrey 1989; Fey 1997). Of course, strategic voting equilibria may involve highly complex computations that go beyond the capabilities of most voters. Behaviorally, therefore, voters may rely on simple voting strategies such as always voting sincerely for the most

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1 These ideas are apparent, for example, in the Merriam-Webster online dictionary (http://www.merriam-webster.com/dictionary/democracy), where ‘democracy’ is defined as: 1 a: government by the people; especially: rule of the majority b: a government in which the supreme power is vested in the people and exercised by them directly or indirectly through a system of representation usually involving periodically held free elections.

2 Many families even vote on important issues like naming a baby. In fact, patents have been issued to protect procedures for making family decisions (http://www.patentstorm.us/patents/7252229/description.html).

3 We do not mean to claim that a “correct aggregation of preferences” in itself maximizes social welfare. It is easy to come up with social welfare definitions that for some distributions of preferences would require choosing an alternative that does not have the most sincere votes. Our goal in this paper is not to derive social welfare conclusions, however.

4 Such behavior is typically referred to as ‘strategic voting’ (Riker 1982a), or ‘tactical voting’ (Galbraith and Rae 1989). In this paper we will use the term ‘strategic voting’. We will refer to voting for one’s most preferred option as ‘sincere voting’.
preferred alternative. In addition, some people may object morally to voting strategically (Lehtinen 2007). In the end, the question whether or not voters vote strategically is an empirical one.

Two examples illustrate situations when strategic voting may occur. First, if the most preferred option does not stand a chance in an election, a voter may vote for her second ranked option in an attempt to avoid even worse outcomes. Such behavior is consistent, for example, with Duverger’s law.\(^5\) Strategic voting may occur, also, if there is a Condorcet loser\(^6\) supported by a minority while a majority is divided between two other alternatives. In this case, sincere voting would give most votes to the minority preferred alternative (Forsythe et al. 1993, 1996; Myatt and Fisher 2002, Gerber et al. 1988). The majority can avoid a victory by the Condorcet loser by coordinating on one alternative. This requires strategic voting by the supporters of one of the two majority alternatives.\(^7\) Our goal is to better understand the occurrence of such strategic voting. We will do so in a combined theoretical and experimental study.

Our study of strategic voting will not include situations with a Condorcet loser, however. Instead, we are interested in a specific environment where we focus on situations where there are Condorcet cycles. In our environment, each of three alternatives (denoted by A, B, and C) has a similar a priori chance of winning the election and each voter faces an a priori symmetric strategic problem. A cycle occurs because sincere voting can lead to any of the alternatives winning if they are voted on sequentially in pair wise votes. Such situations are considered to be widespread (Kurrild-Klitgaard 2001). For example, Neufeld et al. (1994) presents an example of cyclical preferences (and cyclical voting) in the 1925 U.S. Senate. Similarly, Gross (1979) presents an example of cyclical preferences in the 1975 session of the Iowa senate and argues that cyclical voting was finally broken by some senators voting strategically.

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\(^5\) Duverger’s law predicts that in a plurality vote the votes will converge to two candidates, mainly due to the psychological phenomenon of the ‘wasted vote’. Voters supporting a candidate with low perceived chances of winning are assumed likely to move their vote to the more preferred option of the leading two (e.g., Riker 1982b; Palfrey 1989; Fey 1997).

\(^6\) A Condorcet loser is an alternative that would lose any pair wise vote against any other alternative.

\(^7\) Experimental evidence shows that in the absence of coordination devices (such as polls, shared history, previous elections, or ballot position) the Condorcet loser wins a significant proportion of the elections (Forsythe et al. 1993, 1996; for an overview see Palfrey 2006).
An important difficulty in empirically assessing the existence of strategic voting is the fact that it requires knowledge of the voters’ preference orderings over the alternatives. One may try to overcome this by eliciting preferences using survey questions (Cain 1978; Blais and Nadeau 1996; Blais et al. 2001; Myatt and Fisher 2002). Such measurement is subject to noise and strategic reporting, however, both effects which would cloud the analysis. More generally, while analyses using observational data from the field allow one to study the occurrence of strategic voting per se, they do not really allow for a systematic study of its causes and consequences. For this purpose, a controlled laboratory environment is much better suited. Controlled experiments yield suitable conditions to observe behavior with preference orderings clearly defined and known to the experimenter, allowing one to directly observe whether or not a vote is sincere. They also allow for controlled information and direct comparative statics analyses by studying changes in voting when altering one characteristic of the environment at a time.

This is why we use laboratory experiments for our empirical analysis of strategic voting. Before doing so, we will first model the situation as a strategic game and analyze this theoretically. In particular, we will derive Quantal Response Equilibria (QRE) and use these to formulate behavioral predictions. QRE have been show to accurately predict voter behavior before (e.g., Goeree and Holt 2005; Levine and Palfrey 2007; Groß and Schram 2010). QRE has the intuitive advantage that it allows for boundedly rational behavior while at the same time assuming that the error people make declines as the stakes become larger. We will derive such equilibria for the environment we study and show how they predict strategic behavior for voting in groups of various sizes.

The QRE predictions are subsequently tested using our experimental data. Laboratory control will also allow us to measure the impact of changes in the environment on the decision whether or not to vote strategically. Specifically, we are interested in two

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8 For (indirect) empirical evidence of strategic voting in legislatures, see Clinton and Meirowitz (2004). For large electorates Cain (1978) and Myatt and Fisher (2002) provide empirical evidence from the UK and Blais and Nadeau (1996) and Blais et al. (2001) from Canada.

9 Voting studies usually focus on either turnout or candidate choice. With the U.S. and most European countries having voluntary voting, much of the literature considers models adapted to this reality (Bender et al. 2003; Goeree and Holt 2005; Feddersen and Sandroni 2006; Palfrey 2006). In a strategic setting, the turnout decision is often modeled as a participation game (Palfrey and Rosenthal 1983, 1985), where each voter supports either of the two candidates and decides whether or not to cast a vote. In this setting, voting strategically is a dominated strategy. Therefore a voter decides between “abstaining” and “participating and voting sincerely”. As a consequence participation games naturally focus on voter turnout, in particular on the so-called “voter paradox” (Schram and Sonnemans 1996a,b).

10 Quantal response equilibrium (QRE) is a solution concept for games developed by McKelvey and Palfrey (1995). QRE is a generalization of Nash equilibrium that allows for errors in decision making (i.e., boundedly rational behavior). Errors are modeled such that decisions that yield higher expected payoffs are made more frequently than less lucrative decisions.
circumstances that may affect this decision. First, we will study how the relative value attributed to the second preferred option affects voters’ decisions. This is important, because, intuitively, voters are more likely to vote strategically when there is little to lose by having their second option chosen. We can test this directly by comparing elections where the value attributed to the second option winning is close to that of the most preferred alternative to elections where a larger difference between first and second option exists.

Second, we will measure the impact of information about others’ preferences by comparing elections that are identical in all aspects but one, i.e. the fact that these preferences are known in one case and not in the other. This is important, because whether or not voters vote strategically may depend on how much they know about other voters’ preferences. Opinion polls serve to provide such information, which may help voters to coordinate on an alternative and win the election. Voluntary preferences revelation in polls may be strategic, however. In order to isolate the effect of information, we therefore opt for a situation in which an opinion poll truthfully reveals the electorate’s preferences (as in Großer and Schram 2010). Perfect information about the other voters’ preferences will in some of our treatments be made available before the election.

With this information, the decision problem faced by each voter may be even more complex than without. This is because without information all voters face the same a priori situation if every preference ordering is equally likely. Assume for the case with information that supporters of the alternative with the largest support (we call this the ‘majoritarian alternative’) vote sincerely but comprise less than 50% of the electorate. Which voters should then vote strategically? On the one hand, one may think that the supporters of the alternative with the lowest level of support have an incentive to vote strategically to increase their chances. On the other hand, voters for whom the majoritarian alternative is second best may decide to support this to ensure at least this second-best. Whether or not they do so may depend on the relative value they attribute to this option. We will address these issues theoretically and behaviorally in this paper.

When preferences are not revealed by polls, all voters face the same situation. The QRE prediction is then that all voters have the same probability of voting strategically. The

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11 An interesting alternative approach would be to consider sequential elections like the U.S. primaries. Then, information about others’ preferences may be obtained from previous elections. For example, Morton and Williams (1999, 2001) use this to study information asymmetries regarding candidates’ identities.

12 This kind of information can be seen as a ‘noiseless’ version of an early opinion poll, reflecting more the actual preferences than the intended voting behavior. Early opinion polls are not necessarily a good predictor of the actual election outcome. Opinion polls closer to the election tend to capture more strategic behavior and aim basically at predicting the election outcome. For more, see McKelvey and Ordeshook 1986, Gelman and King 1993, Brown and Chappel 1999, or Erikson and Wlezien 2001.
predicted probability of strategic voting in committees is confirmed by our data. With information about the other voters’ preferences, the QRE probability of voting strategically depends on the number of others supporting the same alternative and this alternative’s rank (in terms of support) within the electorate. It also depends on the relative value attributed to the second most preferred alternative. The experimental results are again in line with the QRE predictions. Two important conclusions for the scenario with information are that (i) a higher frequency of strategic voting is observed, the higher is the relative utility of a voter's second most preferred option; (ii) there is coordination on the victory of the majoritarian alternative.

The remainder of the paper is organized as follows: Section 2 presents the model and theoretical analysis, including equilibrium predictions for various sizes of the voting body. The experimental design is introduced in section 3. Section 4 presents and analyzes the results and section 5 presents concluding remarks.

2 The Model
Each of \( N \) voters must choose from three alternatives, \( A, B \) and \( C \). Each voter \( i = 1, \ldots, N \) has a strict preference ordering over these alternatives and will be required to cast exactly one vote. Plurality rule determines the winner, with ties broken by an equal probability random draw. The assumption of mandatory voting allows us to focus on the voting decision without needing to correct for the interaction with the turnout decision. Moreover, the mandatory rule makes strategic voting more salient, since voters are obliged to decide. Mandatory voting exists in many committees and legislators (Nitzan and Procaccia 1986). For national elections, only a minority of countries have mandatory voting (Gratschew, 2001), though it is still prevalent in certain regions, like Latin America.

Voters are assumed to maximize (expected) utility, where a voter’s utility is determined by the rank of the elected alternative in her preference ordering. If her preferred, intermediate or least preferred alternative is elected she receives \( u^b \), \( u^m \) or \( u^l \) respectively. Without loss of generality we normalize by setting \( u^b = 10 \) and \( u^l = 1 \). Then, each voter’s preferences are characterized by \( u^m \), the utility attributed to the intermediate option. Finally, we assume that utility is independent of individuals and options, i.e., \( u^m \) is the same for every voter.\(^{13}\) Hence, only the ordering of the three options distinguishes voters from one another.

We further assume that before an election all voters’ preferences are determined randomly, independently of previous preferences and of other voter’s draws. The own

\(^{13}\) Tyszler (2010) presents results where \( u^m \) varies across voters. The results confirm the conclusions drawn here.
preferences are revealed to the voter by nature before the election. The extent of information about other’s preferences is a variable in the model. The setting can be either uninformed, in which case voters (aside from their own preference ordering) know only the prior probability distribution of preferences, or informed, in which case they know the ex-post realized distribution of preferences for the election concerned. This variable is meant to capture the possible publication of (noiseless) pre-election polls, as described in the introductory section.

An electorate is, therefore, characterized by the number of voters, the distribution of preferences, \( u^m \), and the extent of pre-election information. We define sincere voting as a vote for the most preferred option. A strategic vote is defined as a vote for the second-ranked alternative in the preference ordering (as in Blais and Nadeau 1996; Blais et al. 2001; Cain 1978). The third option, voting for the least preferred option, will only be considered as noisy behavior, because it is a dominated strategy: there is no circumstance under which this could serve the purpose of expected utility maximization.

Because we are most interested in strategic voting caused by the environment and not so much in specific characteristics of the distinct options, we will focus on a game in which every voter has an a priori symmetric problem regardless of his/her preference ordering. We therefore restrict the possible preferences to \( \{(A,B,C); (B,C,A); (C,A,B)\} \), in which the listed order represents the preference ordering. Preferences are independently and randomly drawn from this set with equal probability for each voter. These preferences will typically form a Condorcet cycle, potentially giving rise to strategic behavior. Moreover, there are no Condorcet losers in our setup. We define \( N_{ABC} \) as the number of voters with preference ordering \( (A,B,C) \) (i.e., \( u(A)=10; u(B)=u^m; u(C)=1) \), and similarly \( N_{BCA} \) and \( N_{CAB} \). Note that by construction \( N_{ABC}+N_{BCA}+N_{CAB} = N \). Finally, we denote the election outcome by a vector \( v=(v_A, v_B, v_C) \) such that \( v_A + v_B + v_C = N \), where \( v_k \) denotes the number of votes for option \( k \).

### 2.1 Equilibrium Analysis

Typically, multiple Nash equilibria exist in voting games. Take, for example, a situation in which \( N = 3K \) (\( K \geq 3 \)) and each preference ordering is equally represented (\( K \) voters each) while there is complete information. Then, all situations are Nash equilibria in which voters in exactly two groups vote sincerely and the voters in the remaining group all vote strategically. The election outcome would be, for example, \( v=(0, 2K, K) \) and since no voter is pivotal, no one can benefit from deviating. Another Nash equilibrium is where only one group votes sincerely, with the other two voting strategically. Again, nobody is pivotal. Sincere voting by all may also be an equilibrium. Such voting behavior leads to an expected
payoff of \((u^b + u^m + u^i)/3 = (11 + u^m)/3\). If there are equal numbers of voters for each preference ordering each voter is pivotal, however. Voting strategically will therefore tip the balance to the own second preferred option and yield payoff \(u^m\). As long as \(u^m \leq (11 + u^m)/3\), everyone voting sincerely is a Nash Equilibrium.\(^{14}\) To tackle the multiple equilibria problem one can employ an equilibrium selection device. We will show below, that the equilibrium approach adopted here has as a spinoff that it constitutes such a refinement in the sense that it selects specific Nash equilibria as a special case.

For a variety of political choice problems, a so-called Quantal Response Equilibrium (QRE; McKelvey and Palfrey, 1995) has been shown to be a better predictor of individual choices than Nash equilibrium (Goeree and Holt 2005). For example, Levine and Palfrey (2007) show that it can account for the (seemingly irrational) high turnout rates in large scale national elections, where Nash predicts unrealistically low turnout. To find the QRE for our environment, we start by considering the expected utility derived from voting for distinct options. Consider, for example, voter \(i\) with preference ordering \((A, B, C)\). The expected payoff from voting for option \(A\), denoted by \(u^e_A\), depends on what other voters do. It is a function of the probabilities with which other voters (with the same or other preferences) vote for the three options. Similarly, the expected utility from voting for \(B\) and \(C\), \(u^e_B\) and \(u^e_C\), depend on these probabilities. Nash equilibrium analysis assumes that \(i\) will vote for the alternative that gives her the highest expected utility, \(i.e.,\) she gives the best response to others’ voting probabilities.

In contrast, a QRE analysis allows for the possibility that \(i\) may make an error in deciding what to vote for. One way to allow for error is by adding a stochastic term to the expected utility functions, yielding expected utilities \(u^e_A + \mu \epsilon_A\), \(u^e_B + \mu \epsilon_B\), and \(u^e_C + \mu \epsilon_C\) for options \(A\), \(B\), and \(C\), respectively. In these terms, \(\mu \geq 0\) is an error parameter and the \(\epsilon\) terms are i.i.d. realizations of random variables. This parameterization is general enough to capture different sources of noise, as for example, distractions, perception biases, miscalculations or limited computational capability (Goeree and Holt, 2005).

\(^{14}\) It is also possible to find equilibria in which members of the same group act differently. Take the same example with \(K\) even and \(K \geq 6\). Assume two groups voting sincerely. If the remaining group has half of its members voting sincerely and the remaining voting for their second most preferred alternative the voting outcome could be \(\nu = (K, K/2, 3K/2)\). Once again nobody is pivotal and this is an equilibrium.
A voter will still vote for the option with the highest expected utility but this is now a stochastic event. For example, she will vote for option $A$ if $u'_A + \mu \varepsilon_A > u'_B + \mu \varepsilon_B$ and $u'_A + \mu \varepsilon_A > u'_C + \mu \varepsilon_C$ or
\[
\varepsilon_B - \varepsilon_A < \frac{u'_A - u'_B}{\mu} \quad \text{and} \quad \varepsilon_C - \varepsilon_A < \frac{u'_A - u'_C}{\mu}.
\] (1)

Specification of the distribution functions of $\varepsilon_A$, $\varepsilon_B$, $\varepsilon_C$ yields the probability that $i$ will vote for option $A$ (and similarly for $B$ or $C$). Assuming that the $\varepsilon$’s follow the extreme value type 1 distribution, the (multinomial) probability that $i$ will vote for option $j$, $p'_j$, is given by:
\[
p'_j = \frac{\exp[u'_j / \mu]}{\sum_{l=A,B,C} \exp[u'_l / \mu]}, \quad j = A, B, C.
\] (2)

Next, recall that the probabilities of other voters choosing $A$, $B$, or $C$ enter the expected utility terms in the right hand side of (2). A full specification for all voters then equates a vector of (3N) voting probabilities on the left hand side to a vector of functions of the same probabilities on the right hand side. A QRE (more specifically, a ‘multinomial’ logit equilibrium, MLE) is defined as vector of probabilities that when entered on the right hand side yields itself on the left hand side.

In our framework, the MLE will depend on $\mu$, $u^m$, $N_{ABC}$, $N_{BCA}$, and $N_{CAB}$, as well as on the fact whether or not the latter three numbers are known to the voters. To understand the role of the error parameter $\mu$, note that
\[
\lim_{\mu \downarrow 0} p'_j = \begin{cases} 0, & \text{if } u'_j < \max_k u'_k \\ 1, & \text{if } u'_j = \max_k u'_k \text{ and } u'_j < u'_l, l \neq j \end{cases}
\] (3)
(and $\lim_{\mu \uparrow 0} p'_j$ is $1/K$ if $K$ options ($K=2,3$) yield equal maximum expected utility).

It follows directly from (3) that as noise diminishes to zero, the option with the highest expected utility is chosen, i.e., the MLE converges to a Nash equilibrium (see McKelvey and Palfrey 1995). Similarly,
\[
\lim_{\mu \rightarrow +\infty} p'_j = \frac{1}{3}, \quad j = A, B, C,
\] (4)
which shows that behavior converges to pure randomization as noise increases to infinity.

For any positive and finite value of $\mu$ it is possible to compute MLE. We call the collection of MLE and correspondent $\mu$ values the ‘Multinomial Logit Correspondence’ (MLC). Except for the limit case where $\mu$ approaches infinity, there need not be a unique
MLE. It is possible, however, to identify a unique branch of the MLC that starts from the limit at $\mu = \infty$ and continuously converges to a unique Nash Equilibrium as $\mu \downarrow 0$ (McKelvey and Palfrey, 1995, Theorem 3, item 3). This is called the ‘Principal Branch’ and the corresponding Nash Equilibrium the ‘limiting Multinomial Logit Equilibrium’ of the game.\(^{15}\)

Using the Quantal Response model with the multinomial logit specification has several advantages: (i) it provides a refinement selecting precisely one of the multiple Nash equilibria (i.e., the limiting MLE); (ii) it takes bounded rationality seriously by introducing noise in the individual choice problem; (iii) the principal branch has the intuitive characteristic that players of the same type play symmetric strategies; (iv) in line with intuition, for all finite $\mu$ the MLE probability of choosing an option is increasing in the expected payoff differences with other options. The expected payoff difference will vary with the extent of information and the realized distribution\(^{16}\) but it only includes situations where the voter’s choice makes a difference, since for every non-pivotal situation the payoff difference will be 0.

This last point can be illustrated with an example. As can be easily seen, the right hand side of eq. (2) can be rewritten in terms of expected payoff differences, taking voting sincerely as the reference strategy. For example, for a voter with preference ordering (A,B,C), we write:

$$
\begin{align*}
    p_A^i &= \frac{1}{1 + \exp\left(\frac{(u_B^e - u_A^e)}{\mu}\right) + \exp\left(\frac{(u_C^e - u_A^e)}{\mu}\right)} \\
    p_B^i &= \frac{\exp\left(\frac{(u_B^e - u_A^e)}{\mu}\right)}{1 + \exp\left(\frac{(u_B^e - u_A^e)}{\mu}\right) + \exp\left(\frac{(u_C^e - u_A^e)}{\mu}\right)} \\
    p_C^i &= \frac{\exp\left(\frac{(u_C^e - u_A^e)}{\mu}\right)}{1 + \exp\left(\frac{(u_B^e - u_A^e)}{\mu}\right) + \exp\left(\frac{(u_C^e - u_A^e)}{\mu}\right)}.
\end{align*}
$$

The expected utility difference of voting for option $j$ instead of $k$ $u_j^i - u_k^i$, is a weighted sum of the utility differences between voting for $j$ or $k$ for all possible combinations of votes by other voters (denote by $-i$): $u_j^i - u_k^i = \sum_{-i} P_{-i}(u_j^{-i} - u_k^{-i})$, where $P_{-i}$ denotes the probability that a particular configuration of other voters’ choices occurs and $u_j^{-i}(u_k^{-i})$ gives the expected utility obtained from choosing $j$ ($k$) in situation $-i$. Though there are an extreme number of situations $-i$, for most of them, $i$’s vote will not affect the outcome. In those situations,

\(^{15}\) Except for very special cases, the principal branch needs to be computed numerically. In order to trace it we use the Homotopy Approach as outlined by Turocy (2005, 2010).

\(^{16}\) See appendix A for details of the computations.
so they do not add to the expected utility difference. Therefore, in (5) the voter takes into account only the relevant pivotal situations. An important consequence is that the probabilities in (5) converge to 1/3 as the electorate becomes infinitely large. The intuition is that for infinitely large electorates it no longer matters what any single voter does, and random noise dominates the voter’s choice. We will further discuss this, below. For more details see appendix A.

2.1.1 Uninformed Setting

Consider first the situation without information about other voters’ preferences. The voter knows only the prior distribution of probabilities, the electorate size, the value of the intermediate option and her own preference. Knowing her own preference she can update the probability distribution using Bayes’ rule and use this to calculate the probability of being pivotal given others’ strategies. Subsequently, she can compute her expected payoff differences between voting sincerely, strategically or for the least preferred alternative.

This rather complicated computation is easiest understood by an example. Consider the case in which \( N = 12 \). For a given voter, the most likely distributions among the other voters are \((3, 4, 4), (4, 3, 4) \) and \((4, 4, 3)\), where the first number indicates the number of other voters with the same preference, and the other two the number in the remaining groups. If she believes that all others are voting sincerely this voter considers herself to be pivotal in all three situations (in the first she can create a tie, in the latter two she can break a tie). In the first situation her sincere vote would create a three-way tie and voting strategically would give the victory to her second most preferred candidate. Voting sincerely may be profitable, depending on the value of the intermediate option\(^{17} \). For the other two situations voting sincerely is always a best response, since the voter would be decisive in favor of her most preferred candidate. Considering only these three situations voting sincerely would likely be a best response. In fact, considering all pivotal situations with their respective probabilities it can be shown that voting sincerely is more profitable than voting strategically. In fact, all players voting sincerely constitutes a Bayesian-Nash Equilibrium, regardless of the intermediate preference parameter \( u^m \).

This Bayesian Nash equilibrium is the limiting MLE of the game of incomplete information. Figure 1 shows the corresponding Multinomial Logit Correspondences for three

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\(^{17}\) In this particular situation, voting sincerely will be strictly profitable if \((u^b + u^m + u^s)/3 > u^m \leftrightarrow u^m < 5.5\), where we use the normalization \( u^b = 10 \) and \( u^s = 1 \). Therefore if the intermediate option is low enough, voting sincerely and creating a tie is the best response. If it is high enough, voting strategically is the best response.
sizes of the voting body: \(N=12, 99, \text{ and } 999,999\).\(^{18}\) These are intended to be representative for committees, legislatures, and electorates, respectively. We consider two values for the intermediate option: high \((u^m=8)\) and low \((u^m=3)\).

Figure 1: Multinomial Logit Correspondences for Uninformed Voters

\[\begin{array}{cc}
\text{A} & \text{B} \\
N=12 & N=99 \\
\text{C} & \text{D} \\
N=999,999 & N=999,999 \\
\end{array}\]

**Notes.** Lines show the principle branch of the MLC for high \((u^m=8)\) and low \((u^m=3)\) values of the intermediate option. In panels A, B, and C, the size of the voting body \((N)\) is 12, 99, and 999,999, respectively. Panel D zooms in on the large electorate case for \(\mu \in [0,1]\).

Note that for \(\mu \downarrow 0\), the probability of sincere voting converges to 1 for all \(N\). Hence, for the case of incomplete information (no polls) the limiting MLE is the Bayesian Nash equilibrium without strategic voting, irrespective of \(N\) and \(u^m\). At the other extreme, when noise dominates behavior \((\mu \rightarrow \infty)\), the vote becomes a random choice and voting sincerely, strategically or for the dominated option each occur with probability 1/3. For the intermediate

\(^{18}\) We chose \(N=99\) (999,999) for legislature (electorate) sized voting bodies in order to allow for the possibility of an equal split of preferences.
cases where rationality is somewhat bounded ($\mu \in (0, \infty)$), the MLE probabilities of voting depend on the size of the voting body and on the value attributed to the intermediate option. Previous estimates of $\mu$ using data from voting experiments yield values between 0.4 and 0.8. We will therefore focus some of our discussion on this range of $\mu$-values.

Consider the small (committee size) voting body where $N=12$ shown in panel A. Here the probabilities of voting for the distinct options strongly depend on both $\mu$ and $u^m$. First note that it takes a high value of $\mu$ for voting for the dominated action (not shown in the graph) to be likely. For situations where random noise does not dominate behavior ($\mu < 1$) the probability of voting for the third option is less than 10% and the choice is basically between voting sincerely or strategically. For most levels of noise, the equilibrium level of strategic voting strongly depends on the value attributed to the second preferred option, $u^m$. For $u^m=8$, the probability of voting strategically exceeds 0.25 for a wide range of $\mu$-values. The intuition is that although the limiting (Bayesian Nash) equilibrium is to vote sincerely, one does not lose too much by choosing the second-best. Therefore, an ‘error’ to the best response is not very costly and more likely to occur in the MLE. Focusing on $\mu$-values between 0.4 and 0.8 we observe that the equilibrium probability of a strategic vote is more than three times as high for a high intermediate utility than for $u^m=3$. For $u^m=8$ the model predicts that approximately 30% of the voters will vote strategically for these $\mu$-values.

For legislature-size voting bodies (panel B in figure 1) similar results are obtained, though the MLE probability of choosing the dominated alternative increases to approximately 0.2 for $\mu=1$. Once again, the probability of voting strategically depends strongly on the intermediate utility. For $\mu$ between 0.4 and 0.8 this probability is more or less stable around 0.36 when $u^m=8$ and increases from approximately 0.19 to 0.28 for $u^m=3$. Hence, the equilibrium predicts substantial strategic voting, even in legislature-size groups.

Finally, panels C and D of figure 1 show the multinomial logit correspondences for the probability of voting strategically or sincerely in large electorates (approximately 1 million voters). Here, the probability of being pivotal is so small that the noise term dominates the voters’ decisions. Even for low values of $\mu$, the probability of voting for any of the three options is close to 1/3. Only for values of $\mu < 0.1$ can we distinguish between probabilities for

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19 Goeree and Holt (2005) use data on the participation game reported by Schram and Sonnemans (1996a,b) and find a maximum likelihood estimate of 0.8 for early rounds and 0.4 for late rounds. Tyszler (2008) reports an ML estimate of 0.55 using Brazilian data from a pilot experiment similar to the experiment reported in this paper.

20 For $u^m=8$, for example, when $\mu=1$, the MLE probability of voting sincerely is 0.56 and of voting strategically it is 0.36. Hence, the probability of voting for the dominated option is 0.08.
the distinct options. It is important to note at this stage that it is not our goal to explain strategic voting in large electorates with this model. One could easily adapt the model and arrive at non-random equilibrium probabilities of sincere voting. In the current setup, we conclude that in large electorates significant effects of our model parameters on the probability of strategic voting are only observed for very low levels of noise. In the following analyses we will therefore focus only on committee and legislature size voting bodies.

2.1.2 Informed Setting

Consider next the game with full information. Start with an example with equal share, which can serve as a comparison to the a priori expected situation for uninformed voters in figure 1. Figure 2 plots the principal branch of the MLC for small \((N_{ABC} N_{BCA} N_{CAB}) = (4, 4, 4)\) and medium sized \((N_{ABC} N_{BCA} N_{CAB}) = (33, 33, 33)\) voting bodies. In these cases all voters’ circumstances are again perfectly symmetric. In comparison to the previous case, however, information about others’ preferences removes the uncertainty.

**Figure 2: Multinomial Logit Correspondences for Informed Voters**

Note that in both cases, the Nash equilibrium of sincere voting is still the limiting MLE when the intermediate option is relatively unattractive \((u^m=3)\). For \(u^m=8\), the probability of strategic voting converges as \(\mu \downarrow 0\) to 0.12 for \(N=12\) and to 0.02 for \(N=99\), however. In this case, the

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21 For example, there is no need to assume that error is equally likely for all options. Intuitively, it would seem that the utility gained from the dominated alternative winning is much less prone to noise than the utility of having the most favored alternative win. Similarly, there is no reason why noise in small voting bodies would have the same distribution as in large electorates. Finally, noise need not be the same across voters. Adapting the model in any of these ways could lead to results that differ from those presented here.

22 There is also a practical reason for doing so. Many of the equilibria that follow cannot be computed for very large \(N\).
MLE therefore converges to mixed strategy Nash equilibria with (small) positive probabilities of voting strategically. Otherwise, the results are quite similar to the uninformed case. For the small committee \((N=12)\), the probabilities of voting for the dominated option are small for \(\mu<1\) and large differences in strategic voting are predicted between \(u^m=8\) and \(u^m=3\) when \(\mu\in[0.4,0.8]\). The medium sized legislatures \((N=99)\) are also very comparable. In the informed case, for \(\mu\)-values between 0.4 and 0.8 the MLE probability of a strategic vote is approximately 0.38 when \(u^m=8\) and increases from close to 0 to 0.26 for \(u^m=3\).

Of course, the equal split case is just one of the many distributions of preferences that may be realized (and revealed). In cases where the revealed distribution is unequal, one may expect patterns very different from the uninformed case of figure 1. In an online appendix (available at http://www1.fee.uva.nl/creed/pdffiles/appendixstratvote.pdf) we present the MLC graphs for all possible realizations in the small committee case \((N=12)\). This appendix also includes a table providing for each realization the Nash equilibria selected by the limiting MLE. Figure 3 presents the weighted average of these MLCs, where the weights are given by the probabilities that specific realizations of the preference distribution will occur.\(^{23}\) Therefore, the equilibria in figure 3 may be considered to represent average behavior across multiple committee votes with complete information.

**Figure 3:** Average Multinomial Logit Correspondences for Informed Committees

![Graph showing weighted average MLCs](image)

*Notes.* Lines show the weighted average of the principle branches of the MLCs for high \((u^m=8)\) and low \((u^m=3)\) values of the intermediate option. The average is across all possible combinations of preference orderings, weighted by the probabilities with which they occur.

\(^{23}\) The graph for \(N=99\) cannot be derived due to computational limitations related to the large number of possible preference configurations. The \(N=12\) case is interesting because it represents the case used in our laboratory experiments.
Note that the average of the limiting Nash equilibria across preference configurations is not to vote sincerely. The limiting MLE predicts a weighted average of 73%/76% sincere voting and 24%/22% strategic voting for low and high intermediate value, respectively. Starting with very small $\mu$, the roles are reversed: the MLE predicts more strategic voting when the intermediate value is high. Large differences in strategic voting are predicted between $u^m=8$ and $u^m=3$ when $\mu\in[0.4,0.8]$.

In order to further structure the analysis, a few definitions are useful:

**Definition 1:** The **Majoritarian Set** is the set of alternatives with the highest number of votes if all voters vote sincerely.

**Definition 2:** The **Majoritarian Candidate** is the (set of) alternative(s) with the highest number of votes if voting is restricted to the Majoritarian set and all voters vote sincerely.

Note that if the Majoritarian Set is singleton it equals the Majoritarian Candidate. If it contains two elements (i.e., two options receive equal sincere support, the third receives less) than the Majoritarian Candidate is the option from the Majoritarian Set that gives highest utility to the supporters of the third option. The Majoritarian Candidate is unique, except for the case when all preferences are equally represented, e.g., $(N_{ABC},N_{BCA},N_{CAB}) = (4,4,4)$.

For any distribution of preferences we now first classify voters based on the rank of their most preferred candidate.

**Definition 3:** The **Rank-Type** of a voter is given by:

- **Rank 1st:** Voter whose most preferred candidate is the Majoritarian Candidate.
- **Rank 2nd:** Voter whose most preferred candidate is second in the (sincere) polls.
- **Rank 3rd:** Voter whose most preferred candidate is third in the (sincere) polls.

By ‘sincere polls’ we mean the ranking that occurs if all voters vote sincerely. Duverger’s law suggests that the Rank 3rd voters will be the ones most likely to vote strategically. However, the incentive to do so will depend on the position of the Majoritarian Candidate in their preference ordering. For example, consider $(N_{ABC},N_{BCA},N_{CAB}) = (5,4,3)$. The Majoritarian Candidate is A and the voters with preference ordering CAB are Rank 3rd. If voters with preference ABC vote sincerely, the Rank 3rd voters have no reason to vote strategically.

---

24 We deal with ties as follows. In case all three preference orderings are equally likely, all voters are ranked 1st. If the Majoritarian Set consists of two elements, supporters of the Majoritarian Candidate are ranked 1st, supporters of the other candidate in the set are ranked 2nd and the remaining voters are ranked 3rd. If the Majoritarian Set is singleton and the two other preference orderings have equal support, all voters not supporting the Majoritarian Candidate are ranked 2nd.
because their least preferred candidate will probably not win anyway. In contrast, the Rank 2\(^{nd}\) voters (with preference BCA) may vote strategically in an attempt at a majority coalition with the Rank 3\(^{rd}\). Instead of the Rank-Type, the probability of strategic voting may therefore be determined by the benefits that the Majoritarian candidate, the a priori likely winner, gives to other voters than Rank 1\(^{st}\). We therefore define:

**Definition 4:** The Incentive-Type of a voter is given by:

**(Majoritarian) Supporter:** Voter with the Majoritarian Candidate as the most preferred alternative.

**(Majoritarian) Compromiser:** Voter with the Majoritarian Candidate as the second most preferred alternative.

**(Majoritarian) Opposer:** Voter with the Majoritarian Candidate as the least preferred alternative.

Note that the Rank 1\(^{st}\) and Supporters are by construction the same group. On the other hand Rank 2\(^{nd}\) (Rank 3\(^{rd}\)) can be either Opposers (Compromisers) or Compromisers (Opposers). We can then identify four combination of Rank-Types and Incentives-Types other than rank 1\(^{st}\). Figure 4 plots the weighted average of the Principal Branch of the MLC for these 4 sets.\(^{25}\)

**Figure 4: Strategic Voting by Voter Types**

![Diagram](image)

**Notes.** Lines show the weighted average of the principle branches of the MLCs, distinguishing between high ($\mu^u=8$) and low ($\mu^u=3$) values of the intermediate option in combination with a voter’s Incentive-Type. Rank 2\(^{nd}\) voters are shown in panel A and Rank 3\(^{rd}\) voters in panel B. Only the equilibrium probabilities of voting strategically are shown. The average is across all possible combinations of preference orderings, weighted by the probabilities with which they occur. Cases where groups are tied for Rank 2\(^{nd}\) are not included in the graph (cf. fn 24).

First note that different types play distinct strategies. In the Nash equilibrium (as $\mu \downarrow 0$) Opposers tend to vote strategically. When ranked 3\(^{rd}\) with a low intermediate value, the Nash

\(^{25}\) Separate graphs for each unique situation are available upon request.
equilibrium probability is highest (almost 0.85). Irrespective of rank and intermediate value, Opposers vote more strategically than Compromisers in this limiting MLE.\textsuperscript{26} When there is noise, in particular when $\mu \in [0.4,0.8]$, Rank 3\textsuperscript{rd} voters vote mostly strategically in the MLE.\textsuperscript{27} The Incentive-Type matters as well, however. When $u^m = 3$, Rank 3\textsuperscript{rd} voters are more likely to vote strategically if they are Opposers than if they are Compromisers. The reverse holds for $u^m = 8$.

The latter result is in line with intuition. When they are Compromisers, Rank 3\textsuperscript{rd} voters second choice is the Majoritarian Candidate. A strategic vote is likely to be successful because supporters of this candidate rarely vote strategically. For the high importance of the intermediate option, the benefits of a strategic vote are relatively high. When they are Compromisers, Rank 3\textsuperscript{rd} voters are therefore likely to vote strategically. When they are Opposers, a strategic vote is an attempt to collaborate with the Rank 2\textsuperscript{nd} voters, who themselves are Compromisers. The attraction of a strategic vote is diminished by the fact that the voters it supports are themselves inclined to vote strategically for the Majoritarian Candidate, decreasing the probability of success.

With a low importance of the intermediate option, the interpretation is more complex. First, note that in this case Rank 3\textsuperscript{rd} voters vote less strategically anyway. When Rank 2\textsuperscript{nd} voters are Compromisers, the appeal for a strategic vote is lower than when the importance of the intermediate option is high. Therefore, in equilibrium, they settle less for a compromise, which creates a chance for the Rank 3\textsuperscript{rd} voters (Opposers) to vote strategically by supporting the option most preferred by the Rank 2\textsuperscript{nd} voters. When Rank 3\textsuperscript{rd} voters are Compromisers, Rank 2\textsuperscript{nd} voters are Opposers. A strategic vote by the latter means voting for the option most preferred by the Rank 3\textsuperscript{rd} voters. The incentive for Rank 3\textsuperscript{rd} voters to compromise in this situation is low, especially when together with Rank 2\textsuperscript{nd} voters they have a strong majority over the Supporters. This reasoning implies an increased probability of a strategic vote by Rank 2\textsuperscript{nd} voters (Opposers) and a decreased probability for Rank 3\textsuperscript{rd} voters (Compromisers).

The experiments to be described below study strategic voting in committee-size voting bodies. Based on $\mu \in [0.4,0.8]$, the analysis of the Principal Branch of the MLC yields the following behavioral predictions for $N=12$:

\textsuperscript{26} Not shown in the figure is that in the selected Nash equilibrium, supporters have relatively low probabilities of voting strategically (between 0.05 and 0.17).

\textsuperscript{27} The exception is the group of Rank 3\textsuperscript{rd} Compromisers facing low intermediate value. The MLE for this group is approximately 0.3 for these $\mu$-values.
Behavioral Predictions

1. Without information, the probability of strategic voting is increasing in the importance of the intermediate option (figure 1A).
2. With full information the probability of strategic voting is increasing in the importance of the intermediate option (figure 3).
3. When the value of the intermediate option is low, there will be more strategic voting with information than without (figure 1A vs. figure 3).
4. With full information Rank 3rd voters vote more strategically (on average) than other Rank-Types (figure 4).
5. With full information and a low value for the intermediate option Rank 3rd voters are more likely to vote strategically if they are Opposers than if they are Compromisers (figure 4B).
6. With full information and low value for the intermediate option Rank 2nd voters are more likely to vote strategically if they are Opposers than if they are Compromisers (figure 4A).
7. With full information and low value for the intermediate option, Opposers are more likely to vote strategically than Compromisers (follows from 5 and 6).
8. With full information and high value for the intermediate option Rank 3rd voters are more likely to vote strategically if they are Compromisers than if they are Opposers (figure 4B).
9. With full information and high value for the intermediate option Rank 2nd voters are more likely to vote strategically if they are Compromisers than if they are Opposers (figure 4A).
10. With full information and high value for the intermediate option, Compromisers are more likely to vote strategically than Opposers (follows from 8 and 9).

Our experimental data will allow us to test these MLE predictions. In turn, this will provide an indication of the ability of the MLE to predict strategic voting, which will allow us to better assess its predictions for larger voting bodies.

3 Experimental Design

Twelve sessions were run at the CREED laboratory at the University of Amsterdam, during November and December 2008. 288 student subjects participated, allowing for 24 independent electorates. Each session lasted about one and a half hours. In addition to a show-up fee of €7, subjects were paid €0.05 per experimental point. Average earnings were
€20.46, including the show-up fee. The experiment was computerized using Z-tree (Fischbacher, 2007). Instructions can be found in Appendix B. The experimental design aims at studying the impact on voting behavior of the relative importance of the intermediate option and the extent of information. A full 2x2 combinatorial design therefore requires four treatments. All variations were made across subjects.

The electorate is fixed during a session and consists of 12 voters. Each electorate faces 40 independent elections. In every election, there are three possible preference orderings, \{ (A,B,C); (B,C,A); (C,A,B) \}, which are assigned with equal probability to each subject. There is a new draw before every election. Draws are independent across subjects and elections. Every individual is informed about his or her own preferences before each election. All this is common knowledge. Every experimental electorate experienced the same realization of the random draws (cf. Appendix C), enabling a perfect comparison across electorates.

In every election each subject is required to cast one vote for either A, B or C. Plurality rule determines the winner, with ties broken by equal probability random draw. Subjects are paid in each round according to the rank of the winner in their own preference ordering. If the winner is the highest ranked option a subject is paid 10 points and for the lowest ranked 1 point. The value of the intermediate option is constant for a given electorate and is set to be either 3 or 8 according to the treatment. In the informed treatments, participants know the aggregate induced preferences of all other voters in the electorate in each round, before casting their vote. Specifically, they are told for each of the three preference orderings how many other voters where appointed to it. After each election, the aggregate voting outcome is shown to all subjects. Table 1 summarizes the design.

Table 1: Experimental Design.

<table>
<thead>
<tr>
<th>Information</th>
<th>Intermediate option</th>
<th>Low Importance (u = 3), Uninformed</th>
<th>Low Importance (u = 3), Informed</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>High Importance (u = 8), Uninformed</td>
<td>High Importance (u = 8), Informed</td>
</tr>
</tbody>
</table>

For each cell, we have observations from six electorates. In addition, in August 2007, two pilot sessions were run at Fundação Getúlio Vargas, São Paulo, Brazil.\textsuperscript{28} We used data from this pilot to obtain an out-of-sample estimate of the MLE parameter \( \mu \), for which we found \( \mu = 0.55 \). Using the analysis of section 2, this provides us with specific MLE predictions for our experimental data.

\textsuperscript{28} Two differences with the experiments described here is that the electorate size in the pilot was 15 voters and that the variation in information was made within subjects. Two electorates participated in the pilot.
4 Results

We start with a general overview of voters’ choices and election outcomes in section 4.1. Then, we study in more detail the occurrence of strategic voting across treatments in 4.2 and choices by distinct types in 4.3. In section 4.4 we summarize our findings. Unless indicated otherwise, throughout this section our statistical tests will be non-parametric using average numbers per electorate as units of observation.

4.1 General Overview

For a first impression of the data, table 2 shows for each treatment the distribution of votes across options. Because the labels A, B, and C have no real content, we aggregate votes for most preferred, intermediate, and least preferred option.

<table>
<thead>
<tr>
<th>Intermediate option</th>
<th>Information</th>
<th>Uninformed</th>
<th>Informed</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u^m=3$</td>
<td></td>
<td>1: 0.936</td>
<td>2: 0.049</td>
</tr>
<tr>
<td>$u^m=8$</td>
<td></td>
<td>1: 0.798</td>
<td>2: 0.192</td>
</tr>
</tbody>
</table>

Table 2: Vote Distribution

Notes. Numbers give the fractions of voters in the treatment denoted by the combination of column and row that voted for, respectively, the (1:) most, (2:) intermediate and (3:) least preferred options.

A first thing to note is that we very rarely see subjects voting for the dominated, least preferred option. Second, strategic voting (voting for the intermediate option) is highest (almost 25%) when the intermediate value is high and subjects are informed about the preference distribution. A Kruskal-Wallis test shows that both the fraction of sincere voting and the fraction of strategic voting differ significantly across the four treatments (for sincere voting: $\chi^2=18.12$, $p<0.01$, $N=24$; for strategic voting: $\chi^2=17.49$, $p<0.01$, $N=24$). Pair wise comparisons will follow below. This shows that the combination of information and the value attributed to a voter’s second-best candidate significantly affect the decision whether or not to vote strategically. We will analyze the determinants of strategic voting in more details, below.

Before doing so, we consider the election outcome. In particular, figure 5 shows, across treatments, the fraction of elections where the winner was in the Majoritarian Set or was the Majoritarian Candidate. In case of ties (where a winner was randomly chosen), we did the following. For the Majoritarian Set, if one of the tied options was not in the set and the other was, we counted this as a 0.5 success. For the Majoritarian

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29 Moreover, strategic behavior is not concentrated in a few subjects. A between subjects heterogeneity analysis shows very low variability (the standard deviation is smaller than 0.03), indicating that aggregate average behavior is a good indicator of individual behavior.

30 In case of ties (where a winner was randomly chosen), we did the following. For the Majoritarian Set, if one of the tied options was not in the set and the other was, we counted this as a 0.5 success. For the Majoritarian
Set and the Majoritarian Candidate are better predictors of the election outcome when the intermediate value is low. This is in line with intuition because behavioral prediction 1 is that less strategic voting is to be expected when $u^m=3$ which in turn will improve the chances of the Majoritarian Candidate. Moreover, information leads to strong coordination around the Majoritarian Candidate, which wins over 93% of the elections.

**Figure 5: Majoritarian Set and Majoritarian Candidate**

![Graph showing the fraction of election outcomes for each treatment.](image_url)

*Notes.* Bars show for each treatment the fraction of election outcomes that are, respectively in the Majoritarian Set or equal to the Majoritarian Candidate.

### 4.2 Strategic Voting

For each treatment figure 6 shows the fraction of strategic votes across rounds. It also includes the MLE predictions based on the value for $\mu$ estimated with the pilot data (*cf.* section 3). A first, general, impression from the figure is that the MLE predictions for low intermediate values fare quite well. For both the uninformed and the informed cases, the data are close to the prediction. For high intermediate values, the observations appear to be somewhat lower than predicted.

Comparing across treatments, we observe more strategic voting when the intermediate value is high than when it is low. The difference is statistically significant for both informed and uninformed voters (in both cases, $MW, Z=−2.882, p<0.01, N=12$). This is in support of behavioral predictions 1 and 2 of section 2. In short, even though the observed extent of

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Candidate we counted the winner as 0.5 in case we observed a tie with one other candidate and 0.33 in case of a three-way tie.
Figure 6: Uninformed Setting: Experimental Data and Predictions

A: Uninformed

B: Informed

Notes. Lines show the 3-period moving average of the fraction of strategic votes in the uninformed (panel A) and informed (panel B) sessions. Dashed (solid) lines refer to low (high) intermediate values. Light lines show the 3-period moving average MLE predictions. Note that in the informed case (panel B) the MLE prediction in a round depends on the realized distribution of preferences.

strategic voting when \( u^m = 8 \) is somewhat lower than predicted by MLE, the comparative static prediction that it is higher than for \( u^m = 3 \) does find (strong) support in our data.

A comparison of panels A and B shows whether information affects strategic voting. For low intermediate value, information is predicted to boost strategic voting (cf. behavioral prediction 3). More specifically, MLE predicts the average fraction of strategic votes to be 0.06 and 0.16, respectively, for uninformed and informed voters. We observe fractions equal to 0.05 when voters are uninformed and 0.17 when they are informed (cf. table 2). In support
of behavioral prediction 3, the observed increase is statistically significant ($MW, Z=-2.882, p<0.01, N=12$). For high intermediate values, we observe on average 0.19 and 0.25 of the voters doing so for the uninformed and informed cases, respectively, where 0.31 is predicted for both cases. This difference is not statistically significant ($MW, Z=-1.761, p=0.09, N=12$).

### 4.3 Voter Types

Next, we consider the variation of strategic behavior across voter types (i.e., Rank-Types and Incentive-Types) distinguished in section 2. Figure 7 shows this for the treatment with information (without information, voters do not know their own type, of course). We clearly observe that Rank 3$^{rd}$ voters are most likely to vote strategically. In fact, they vote more often strategically than sincerely. In contrast, Rank 1$^{st}$ voters basically never vote strategically and Rank 2$^{nd}$ voters vote strategically often, but less than half of the time.

**Figure 7: Strategic Voting and Voter Types**

![Figure 7: Strategic Voting and Voter Types](image)

**Notes.** Bars show for the informed treatment the fraction of votes that were strategic. Voter types are distinguished along the horizontal axis and the intermediate value treatments by the color of the bar.

When the intermediate value is low, the difference between strategic voting of Rank 3$^{rd}$ types and Rank 2$^{nd}$ types is statistically significant (Wilcoxon ($W$) signed rank tests: Rank 3$^{rd}$ Opposers vs. Rank 2$^{nd}$ Opposers and Rank 3$^{rd}$ Compromisers vs. Rank 2$^{nd}$ Compromisers both have $Z=-2.201, p=0.03; N=6$). Moreover, both Rank 3$^{rd}$ and Rank 2$^{nd}$ types vote strategically significantly more often than Supporters do (in both cases $W, Z=-2.201, p=0.03; N=6$). The exact same results are obtained for the treatment with high intermediate value.$^{31}$

$^{31}$ The reason why the test statistic always has the same value is that in all tests, the ranks are unanimous across the 6 electorates for any comparison. 6 out of 6 positive ranks gives $Z=2.201$ and $p=0.028$ in the Wilcoxon test.
These results provide strong support for the fourth behavioral prediction that Rank 3\textsuperscript{rd} voters are most likely to vote strategically.

The fifth and sixth behavioral predictions relate to the case with low intermediate value and predict that, respectively, Rank 3\textsuperscript{rd} and Rank 2\textsuperscript{nd} voters will vote more strategically if they are Opposers than if they are Compromisers. Because in two out of six electorates we observe Rank 3\textsuperscript{rd} Compromisers voting more strategically than Rank 3\textsuperscript{rd} Opposers, we cannot support behavioral prediction 5 ($W, Z=-0.314, p=0.75, N=6$). For the same comparison with Rank 2\textsuperscript{nd} voters, we observe only one electorate with the right sign, though this is not enough to create a statistically significant test result in the wrong direction ($W, Z=-1.572, p=0.12, N=6$). While the Incentive-Type does not appear to have an effect when considered in interaction with the Rank Type, it may when viewed in isolation. Behavioral prediction 7 is that Opposers will vote more strategically than Compromisers for low intermediate value. We observe the opposite, however: less strategic voting by Opposers (0.29) than by Compromisers (0.44). The difference is statistically significant ($W, Z=-1.992, p=0.05, N=6$), a clear rejection of prediction 7. This is no surprise, since behavioral prediction 7 is a direct consequence of behavioral predictions 5 and 6 which were not supported by our data.

Next, the eighth and ninth predictions are that for high intermediate value, respectively, Rank 3\textsuperscript{rd} and Rank 2\textsuperscript{nd} voters will vote more strategically when they are Compromisers than when they are Opposers. In both cases, the prediction is supported in five out of six electorates. For Rank 3\textsuperscript{rd} voters, this is not enough to render statistical significance ($W, Z=-1.153, p=0.25, N=6$). For Rank 2\textsuperscript{nd} voters, the difference is statistically significant, however ($W, Z=-1.992, p=0.05, N=6$). Finally, behavioral prediction 10 is that Compromisers will vote more strategically than Opposers for high intermediate value. This is indeed observed in our data, where the fraction of strategic votes is 0.63 and 0.38, respectively. The difference is statistically significant ($W, Z=-2.201, p=0.03, N=6$), in support of the prediction.

All of the previous tests have been univariate and based on average results per electorate. To increase the power of the tests we can consider the data as deriving from a panel where every participant votes in 40 elections. We do so by conducting a probit regression explaining the individual choice to vote at a particular election, with random effects at the electorate level. Table 3 presents the estimated marginal effects. We have added a variable indicating the period number (divided by 10) to see if any learning is taking place (which there is not). We also added a dummy variable indicating elections where one of the options was supported by an absolute majority, because in this case strategic voting may simply be seen as futile.
This was the case in 27.5% of the elections. The results show that the probability of voting strategically is 3.5 percentage points lower in these rounds, when the intermediate value is high (the effect for low value is statistically insignificant). Other factors remain important, however.

**Table 3: Strategic Voting**

<table>
<thead>
<tr>
<th>Intermediate Value</th>
<th>Low ($u^m=3$)</th>
<th>High ($u^m=8$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant (coefficient)</td>
<td>$-2.677^{**}$</td>
<td>$-1.947^{**}$</td>
</tr>
<tr>
<td>Period/10</td>
<td>0.006</td>
<td>0.003</td>
</tr>
<tr>
<td>Simple Majority</td>
<td>0.013</td>
<td>$-0.035^{*}$</td>
</tr>
<tr>
<td>Compromiser</td>
<td>0.148</td>
<td>0.306^{**}</td>
</tr>
<tr>
<td>Opposer</td>
<td>0.079</td>
<td>0.207^{**}</td>
</tr>
<tr>
<td>Rank 2^{nd}</td>
<td>0.207^{**}</td>
<td>0.247^{**}</td>
</tr>
<tr>
<td>Rank 3^{rd}</td>
<td>0.542^{**}</td>
<td>0.567^{**}</td>
</tr>
<tr>
<td>Rank 2^{nd} x Opposer</td>
<td>0.002</td>
<td>$-0.056$</td>
</tr>
<tr>
<td>Rank 3^{rd} x Opposer</td>
<td>0.059</td>
<td>0.019</td>
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Notes. The table presents the results of a random effects probit regression model where the dependent variable is a dummy indicating whether or not voter $i$ in electorate $j$ voted strategically in election $t$. Formally, it gives the marginal effects derived from the regression model $\Pr(t_i) = \Phi(X' \beta + \mu_i)$ where $\Phi$ gives the probability that $i$ of $j$ votes strategically in $t$. $\Phi$ denotes the cumulative normal distribution and $X$ is the vector of independent variables described in the first column of the table. $\mu_i$ is a (white noise) electorate-specific error that corrects for the dependencies across individual decision in the same group. The independent variable ‘Simple Majority’ is a dummy variable indicating situations where one of the preference orderings had an absolute majority of at least 7. The independent variables with an ‘x’ between variables indicate interaction terms. To avoid the dummy trap, the variable indicating Rank 1 voters (i.e., Supporters) has been left out of the regression. The tests depicted in the last two rows test equality of the estimated coefficients. Our results are not sensitive to the choice of quadrature points; when varying these points all differences are smaller than $10^{-9}$.

* (***) denotes statistical significance at the 5% (1%)-level.

The results also show that, irrespective of the intermediate value, Rank 2^{nd} and Rank 3^{rd} voters are both more likely to vote strategically than Supporters (the category absorbed in the constant term), and that Rank 3^{rd} voters are most likely to vote strategically. This confirms the results from our univariate analyses. In fact, Rank 3^{rd} voters have a 54-57%-points higher probability of voting strategically than Supporters. The effect of incentive-type is lower. With low intermediate value it does not matter statistically whether one is Supporter, Compromiser, or Opposer. When $u^m=8$, both Compromisers and Opposers vote strategically more often than Supporters do, but the difference between the two is statistically insignificant. Compromisers have a 31%-point higher probability of voting strategically than Supporters do.

The results for the interaction between Rank-type and Incentive-Type, by and large, again support the conclusions from the univariate analyses. One difference is that for $u^m=8$, the dif-
ference between Rank-2\textsuperscript{nd} Compromisers and Opposers is now statistically significant. Note that this effect has the opposite sign than that predicted by Behavioral Prediction 6, however.

4.4 Summarizing the Results
In general, the Quantal Response model predicts behavior in the experimental setting quite well. Data support the behavioral predictions at the aggregate level as well as the comparative statics for our treatments (which vary information and intermediate value). In particular, we observe that (i) with and without information, the probability of strategic voting is increasing in the importance of the intermediate option; (ii) with low intermediate value, there is more strategic voting with information than without; (iii) there is no statistically significant effect of information when the intermediate value is high.

When considering behavior disaggregated per type of voter, the predictive power of the MLE decreases. Our data do support the prediction that (iv) with full information Rank 3\textsuperscript{rd} voters are more likely to vote strategically than Rank 2\textsuperscript{nd} voters (and both more often vote strategically than Rank 1\textsuperscript{st} do). Moreover, we observe that (v) Compromisers vote more strategically than Opposers and both more than Supporters, MLE predicts this comparative statics only for high intermediate value. MLE does predict specific patterns for combinations of voter ranks and voter types (Behavioral Predictions 5-10) but we find only limited support for these detailed predictions.

5 Concluding remarks
We have studied a voting environment characterized by the regular occurrence of Condorcet cycles in preferences. Voters are faced with the decision of voting sincerely or strategically. Voters know their own preference, but may or may not have information about the distribution of preferences across the electorate. When this information is available, certain characteristics of this distribution (such as the rank of the support for one’s most preferred candidate or the relative position of the plurality-preferred candidate in one’s preference ordering) may become important elements in determining what to vote for. The way such factors affect the probability of voting strategically is captured quite well by the predictions derived by adding bounded rationality to a standard utilitarian voting model and deriving the Multinomial Logit Equilibrium (MLE).

Our goal has been to establish whether or not people vote strategically, and what factors affect the probability of doing so. For this purpose we excluded one of the obvious candidates for strategic voting, i.e., situations with a Condorcet loser. Instead, we have created an environment in which options are a priori symmetric and where Condorcet cycles are likely
to occur. In this environment, one that is regularly observed in the field, a strategic vote aims at securing one’s second-preferred option as opposed to trying to have the most-preferred option win. Our boundedly-rational equilibrium model (the Quantal Response Equilibrium) allowed us to derive theoretical predictions on strategic voting. In this way, the equilibrium analysis provides an important tool for understanding the strategic vote. In the end, whether or not voters vote strategically is an empirical question, however. For this, our experiments have proved to be important. Laboratory control has allowed us to provide precise answers to this question. We know exactly when a subject in the experiment votes strategically and when she does not. By varying model parameters one at a time, we have been able to trace the causality between changes in these parameters and the vote. This has allowed us to establish that voters vote more strategically when the relative value of the second-preferred option increases but that knowing the distribution of preferences makes strategic voting more likely if this relative value is low.

Laboratory control has also allowed us to study in detail who votes strategically. We find strong evidence for the (intuitive) MLE prediction that voters who prefer the candidate with the largest support (the ‘Majoritarian Candidate’) will sincerely vote for this candidate. For strategic voting by other voters, there are two characteristics of their preferences that may play a role. First, it may matter whether a voter has the Majoritarian Candidate as a second or third preferred option. In the former case, she may decide to vote strategically in an attempt to help the supporters of this candidate to obtain a majority. Second, it may matter how a voter’s most preferred candidate ranks in a poll where everyone votes sincerely. If this rank is lowest in the polls, the voter may vote strategically believing that her most preferred candidate does not stand a chance. Our data show that the second argument is more important than the first. Though a voter’s personal ranking of the Majoritarian Candidate does affect the probability of voting strategically, the ranking of one’s most preferred candidate in the electorate is more important. Compared to a supporter of the Majoritarian Candidate, a supporter of the lowest ranked candidate has a more than 50%-point higher probability of voting strategically.

Of course, a downside of using laboratory experiments is that we were forced to restrict the analysis to committee-size voting bodies. The confirmation of the main MLE-predictions for committees does give some confidence in their predictions for larger voting bodies, however (see Levine and Palfrey 2007 for a similar argument with respect to voter turnout experiments). Moreover, there is ample empirical evidence (reviewed in section 1 and footnote 8) that substantial strategic voting takes place even at national elections. Hence, the
question is not whether strategic voting takes place, but what the causes and effects of strategic voting are. Our design and results pertain to this question.

In the introduction to this study, we argued that a sufficient condition for correct aggregation of preferences is that every voter casts a vote for her most preferred alternative. If this occurs, the winner is in the set of Majoritarian Candidates 100% of the time. In our experiment, we observe this 72-88% of the time with uninformed voters and 93-96% of the time when voters know each others’ preferences (cf. figure 5). From our laboratory results, we therefore conclude that opinion polls revealing the distribution of preferences are sufficient for voting to correctly aggregate preferences in this way. With such opinion polls, the plurality’s desire is usually honored, even when some voters vote strategically. We conclude that in both our theory and experiments information works as a coordination device around the victory of the Majoritarian Candidate. Summarizing our answer to the main research question, information impacts voting behavior by increasing strategic behavior in some situations, differentiating voting patterns across types, and promoting a higher chance of victory for the Majoritarian Candidate.

32 Of course, if this is the case, it may create an incentive to strategically misreport preferences in a poll. This is a topic that can easily be studied in future experiments.
References


Appendix A

In this appendix we show how the probabilities of voting for various options depends on the probabilities of being pivotal in various situations and how this yields the conclusion that these probabilities converge to 1/3 as the size of the electorate increases to infinity.

The multinomial probability that a voter \( i \) with preference ordering (A,B,C) will vote for option \( j=A,B,C \), is denoted by \( p_j^i \), and given by eq. 5, which we summarize by

\[
p_j^i = \frac{\exp\left[\left(u_j^i - u_k^i\right)/\mu\right]}{1 + \exp\left[\left(u_j^i - u_k^i\right)/\mu\right]} + \exp\left[\left(u_k^i - u_j^i\right)/\mu\right], \quad j=A,B,C
\]

(A1)

Recall that the expected utility difference of voting for \( j \) instead of \( k \), \( u_j^i - u_k^i \) is a weighted sum of the utility differences between voting for \( j \) or \( k \) for all possible combinations of votes by other voters (−\( i \)). For example:

\[
u_A^i - u_B^i = \sum_{j=1}^N P_{ij}(u_A^i - u_B^i)
\]

(A2)

where \( P_{ij} \) denotes the probability that a particular configuration of other voters’ choices occurs and \( u_j^i(u_k^i) \) gives the utility obtained from choosing \( j \) (\( k \)). A configuration of other voters’ choices depends on their preferences and on their choices conditional on their preferences.

There are \( \binom{N+1}{2} \) possible preference configurations for other voters. Each will take the form \((N-I_{ABC}, N-I_{BCA}, N-I_{CAB})\), and will occur with (multinomial) probability:

\[
P_{(N-I_{ABC}, N-I_{BCA}, N-I_{CAB})} = \frac{(N-1)!}{(N-1_{ABC})!(N-1_{BCA})!(N-1_{CAB})!}\left(\frac{1}{3}\right)^{(N-1)}
\]

(A3)

In each of these, the probabilities of various configurations of the others’ votes depends on their strategies, i.e., the probabilities with which they vote for A, B, or C. These then determine the probabilities that \( i \) will be pivotal. For all non-pivotal situations, \( u_A^i = u_B^i = u_C^i \). It follows directly from (A2) that only pivotal probabilities are relevant in determining the expected utility differences in (A1).

To illustrate, consider the configuration of other voters preferences \((N-1,0,0)\), i.e, all other voters have preference ordering (A,B,C), which occurs with probability \( P_{(N-1,0,0)}=(1/3)^{N-1} \). For simplicity, consider only quasi-symmetric strategies. 33 One of the pivotal situations faced by a voter with preference (A,B,C) is a tie between A and B. This occurs with probability:

\[
P_{A=B|(N-1,0,0)} = \sum_{i=1}^{\left\lfloor \frac{N-1}{2} \right\rfloor} \frac{(N-1)!}{i!i!(N-1-2i)!}\left(p_{A,B,C}^{(A,B,C)}\right)^i\left(p_{B,C}^{(A,B,C)}\right)^i\left(p_{C}^{(A,B,C)}\right)^{(N-1-2i)}
\]

(A4)

where \( P_{A=B|(N-1,0,0)} \) denotes the probability that a tie occurs between A and B conditional on the distribution of others’ preferences being \((N-1,0,0)\), and \( \left\lfloor x \right\rfloor \) (\( \langle x \rangle \)) indicates the rounding up (down) of \( x \). Note that the sum is restrained to consider only situations where C receives fewer

33 Quasi-symmetric strategies are strategies that are equal for all players with the same preferences and information and facing the same environment (e.g. Palfrey and Rosenthal 1983).
votes than $A$ and $B$ or a three-way tie, i.e., a vote for $A$ is decisive in favor of $A$ while a vote for $B$ is decisive in favor of $B$.

One can derive pivotal probabilities as in (A4) for all configurations of voter preferences and strategies and substitute them for $P_{-i}$ in (A2) (whilst neglecting all $P_{-i}$ for non-pivotal situations). Note that as $N$ increases each pivotal probability as in (A4) converges to 0. As a consequence, the difference in expected utility in (A2) converges to 0 and the probability of voting for any specific option in (A1) converges to 1/3.
Appendix B

In this appendix we provide a transcript of the instructions for the treatment with high intermediate value and full information. The paragraph denoted in italics was omitted in the treatment without information. Note that there were 24 subjects in the laboratory in any given session.

Welcome

Welcome to this experiment in decision making. Please read these instructions carefully. They will explain the situations you will be facing and the decisions you will be asked to make.

In this experiment you will earn money, which will be paid to you privately at the end of the session. Your earnings will depend on your decisions as well as on the decisions of other participants in today’s experiment.

Your earnings in the experiment will be in experimental “points”. At the end of the experiment, each experimental point will be exchanged for euros at a rate of €0.05 per point. For example, if you earn 200 points, your earnings will be €10. In addition, you have already received €7 for showing up on time.

Rounds and Decisions

In this experiment, you will play various rounds. The total number of rounds will not be revealed, however. In each round, you will be asked to make exactly one decision.

Your decision in any round consists in voting for one of the options: A, B or C. The electorate consists of 12 people whose identities will not be revealed. This electorate will be kept fixed during the whole experiment. Each member of the electorate will have the same three options to vote from.

The option elected will be the one receiving the highest number of votes (out of 12). In case of a tie, one of the options with the highest number of votes will be randomly selected with equal chance.

Your Preference Ordering

In each round you will be assigned a preference ordering which will determine your earnings according to the winner of the vote.

Your preference ordering, and the preference ordering of your colleagues, can be one of the following:

A B C
B C A
C A B

In case the elected option is the option listed first you will receive 10 points;
In case the elected option is the option listed second you will receive 8 points;
In case the elected option is the option listed last you will receive 1 point.

In each round, each of the 3 preference orderings will be attributed to each person independently with equal chance. Therefore, your preference ordering will often change from one round to another. Before you cast your vote, you will be informed of your preference ordering for that round. We advise that at the start of every round you take a moment to check this preference ordering.

In addition, at the start of very round, you will be informed how many participants in your electorate have been attributed to each of the three preference orderings. For example, you may hear that 5 voters have preference ordering A B C, 3 voters have B C A and 4 voters have C A B. In addition, you will also know your own preference ordering for the round, of course.
**Trial Round**

Before we start with the actual experiment, there will be one trial round at the start of the experiment. This trial round proceeds in exactly the same way as the rounds in the experiment itself, but it will have no consequences for actual earnings.
Appendix C

Table C1 shows the realizations of the random draws for the preference distributions for the 40 elections. The same realizations were used in all electorates.

Table C1: Realized Preference Distributions

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<th>Majoritarian Candidate</th>
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